THIRD EDITION

PRINCIPLES OF ELECTRIC MACHINES AND POWER ELECTRONICS

P.C. SEN

WILEY

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WILEY

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In loving memory of my parents,
I dedicate this book to my loving and wonderful family

About the Author

Dr. Paresh C. Sen is Emeritus Professor of Electrical and Computer Engineering at Queen's University, Canada. Dr. Sen received his Ph.D. degree at the University of Toronto in 1967. He has worked for industries in India and Canada and was a consultant to electrical industries in Canada. He has authored over 215 technical papers in the general area of electric motor drives and power electronics. He is the author of two internationally acclaimed textbooks: *Principles of Electric Machines and Power Electronics* (Wiley, 1989, 1997, 2013) and *Thyristor DC Drives* (Wiley, 1981). He has taught electric machines, power electronics, and electric drive systems for over 45 years. His fields of interest include power electronics, electric drive systems, switching power supplies, wind energy systems, digital control, fuzzy logic control, and modern control techniques for power electronics and motor drive systems.

Dr. Sen has served IEEE in various capacities: as an associate editor, distinguished lecturer, chairman of the technical committees on power electronics and energy systems, conference session organizer, conference session chairperson, society committee member, and paper reviewer. He served as an NSERC (Natural Science and Engineering Research Council of Canada) Scientific Liaison Officer evaluating university–industry-coordinated projects.

Dr. Sen is globally recognized as an authority in power electronics and motor drive systems. He received the IEEE–IAS (Industry Application Society) Outstanding Achievement Award in 2008 with a prize money of USD5000, and the IEEE–Canada Outstanding Engineering Educator Award in 2006 for his seminal contributions over four decades as a researcher, supervisor, teacher, author, and consultant. Dr. Sen received the IAS–IDC Prize Paper Award in 1986. He is a fellow of IEEE and a fellow of EIC. As an Emeritus Professor, Dr. Sen continues to be active in research and in several IEEE societies.

PREFACE TO THE THIRD EDITION

There has been widespread use of electric machines in industries and household appliances. New applications of electric machines have emerged. Novel control strategies, new circuit topologies, and new solid-state devices have contributed to further performance improvements of electric machine systems. This edition is an attempt to update the contents of the book, and to respond to the suggestions of readers and instructors. To these ends the following new material has been incorporated.

- Electric machines are mostly used as motors, and in most books the motoring mode of operation is extensively discussed. In recent years there has been phenomenal worldwide industrial growth of wind energy generation. These systems use electrical machines as generators. A chapter (Chapter 11) on wind energy systems has therefore been added for better understanding of the use of electric machines in their generating mode of operation.
- In electric motor drive systems, sensors have been used to feed back speed and/or position information. These sensors have several disadvantages, such as less reliability, less robustness, more maintenance requirements, and more cost. Recently drives without sensors have been used in industry. The concept of sensorless drives has been presented in Chapter 6. As well, the coverage on permanent magnet motor drives and brushless dc drives has been updated in Chapter 6.
- The coverage of solid-state devices, such as Power MOSFETs, IGBTs, and GCTs, has been updated in Chapter 10.
- Power supply in computers, datacenters, servers, and so forth, is an important application of power electronic technology. Introductory coverage of high-power density power supply for such applications is presented in Chapter 10. As well, the popular buck converter configuration for power supply is presented in Chapter 10.
- Neutral point clamped (NPC) inverters are gaining popularity for high-power and high-voltage applications. The concept of NPC inverters is presented in Chapter 10.
- The book presently has a large number of problems and examples. Additional problems and examples have been added to chapters and sections that appear to have been used by most instructors. In the second edition of my book, very few problems dealt with simple, basic understanding. Therefore, most of the problems that have been added for the third edition are designed to test the basic understanding of the concepts covered in the book.

I am overwhelmed by the widespread acceptance of my book and the many complimentary comments I have received from individuals: students, professors, engineers, and research workers. Writing this book has given me much satisfaction and a sense of achievement. I consider my book as my best contribution to education. This book will be my legacy.

I am grateful to my graduate students, Zhiyuan Hu, Christopher Fiorentino, Wilson Eberly, Liang Jia, Zhiliang Zhang, Jizhen Fu, among others, for their valuable assistance. Zhiyuan drew the figures for the third edition. Christopher assisted me in checking and editing the draft copy of the manuscript of the third edition.

I thank Kendra Pople-Easton and Mary Gillespie for secretarial assistance. I thank my department heads, Dr. Steven Blostein and Dr. Michael Greenspan, for allowing me to continue to use my office, and for their continued encouragement in this venture.

I thank my former Ph.D. student, Dr. Chandra S. Namuduri, whom I consider an expert on electric machines, for having useful discussions on industrial applications of electric machines. I thank my colleague, Dr. Yan Fei Liu, and my friend, Dr. Bin Wu of Ryerson University, with whom I had valuable discussion on the contents of this revision. I thank my other colleagues who treated me with respect, and expressed interest in the book. I express my profound gratitude to my mentor and friend, Chuck (Prof. Charles H. R. Campling), who again spent many hours reading and correcting the text.

I thank Mr. Dan Sayre, editor and associate publisher of John Wiley & Sons, Inc., for his cooperation, encouragement, and understanding. It has been a pleasure working with him.

Last but not least, I thank my wife Maya, and my children, Sujit, Priya, and Debashis, for their continued support throughout the endeavor.

One more person I want to thank. He was my grade four teacher in Bangladesh. In 1972, right after Bangladesh became an independent country, I visited the village, Haidgaon, in Chittagong, Bangladesh, where I was born. There, my grade four teacher came to see me. This was our conversation:

Teacher: Paresh, I hear you live in Canada.

I: Yes, I do.

Teacher: What do you do in Canada?

I: I am a professor at a Canadian University.

Teacher: Oh, you are a professor! How many books have you written?

I: None.

Teacher: None! What do you do then?

His comment started to bother me, and so I decided to write a book. In fact, I ended up writing two books: (1) *Thyristor DC Drives* (Wiley, 1981) and (2) *Principles of Electric Machines and Power Electronics* (Wiley, 1989, 1997, 2012). I am thankful to my grade four teacher, whose comment inspired me to write books.

Queen's University
Kingston, Ontario, Canada
April 2012

P. C. SEN

PREFACE TO THE SECOND EDITION

Technology never stands still. Since the first edition of this book, there have been new developments in the applications of, for example, permanent magnet motors and solid-state devices for control. The basics of electric machines and machine control remain the same, however. Thus, preserving the content of the first edition, which has had widespread acceptance, this edition endeavors to enhance, to update, and to respond to the suggestions of readers and instructors. To these ends, the following new material has been incorporated.

- A large number of new problems and some new examples have been added. Most of these problems are presented in the chapters and sections that appear to have been used by most instructors. The number of problems in the second edition is nearly double the number in the first edition.
- Coverage of permanent magnet motors has been introduced, including permanent magnet dc (PMDC) motors, printed circuit board (PCB) motors, permanent magnet synchronous motors (PMSM), brushless dc (BLDC) motors, and switched reluctance motors (SRM).
- Constant-flux and constant-current operation of induction motors is discussed.
- Additional material is included on new solid-state devices, such as insulated gate bipolar transistors (IGBT) and MOS-controlled thyristors (MCT). This material appears in Chapter 10. This chapter also includes, for the first time, material on Fourier analysis of waveforms, current source inverters using self-controlled solid-state devices, and three basic configurations of choppers.
- A concise treatment of three-phase circuits is presented in Appendix B.
- Answers to some problems are presented in Appendix E to assist students in building confidence in their problem-solving skills and in their comprehension of principles.

Many individuals have expressed their opinions on the first edition and have made suggestions for the second edition. I acknowledge with gratitude these contributions, as well as the generous comments of many who have written and spoken to me—students, instructors, and research workers. The number is so large that it would be inappropriate to name them all and the risk of omission would be great.

I am grateful to my graduate students, Yan Fei Liu and Zaohong Yang, for their valuable assistance. I thank the departmental secretary, Debby Robertson, for typing the manuscript of the second edition at various stages, Jennifer Palmer and Patty Jordan for secretarial assistance, and Perry Conrad, the departmental manager, who made the administrative arrangements. I thank my wife Maya and my children, Sujit, Priya, and Debashis, who were a constant and active source of support throughout the endeavor. Last but not least, I express my profound gratitude to Chuck (Prof. C. H. R. Campling), who again spent many hours reading and correcting the text. His friendship, valuable counsel, and continued encouragement are greatly appreciated.

Queen's University
Kingston, Ontario, Canada
January 1996

P. C. SEN

PREFACE TO THE FIRST EDITION

Electric machines play an important role in industry as well as in our day-to-day life. They are used in power plants to generate electrical power, and in industry to provide mechanical work, such as in steel mills, textile mills, and paper mills. They are an indispensable part of our daily lives. They start our cars and operate many of our household appliances. An average home in North America uses a dozen or more electric motors. Electric machines are very important pieces of equipment.

Electric machines are taught, very justifiably, in almost all universities and technical colleges all over the world. In some places, more than one semester course in electric machines is offered. This book is written in such a way that the instructor can select topics to offer one or two semester courses in electric machines. The first few sections in each chapter are devoted to the basic principles of operation. Later sections are devoted mostly to a more detailed study of the particular machine. If one semester course is offered, the instructor can select materials presented in the initial sections and/or initial portions of sections in each chapter. Later sections and/or later portions of sections can be covered in a second semester course. The instructor can skip sections, without losing continuity, depending on the material to be covered.

The book is suitable for both electrical engineering and non-electrical engineering students.

The dc machine, induction machine, and synchronous machine are considered to be basic electric machines. These machines are covered in separate chapters. A sound knowledge of these machines will facilitate understanding the operation of all other electric machines. The magnetic circuit forms an integral part of electric machines and is covered in Chapter 1. The transformer, although not a rotating machine, is indispensable in many energy conversion systems; it is covered in Chapter 2. The general principles of energy conversion are treated in Chapter 3, in which the mechanism of force and torque production in various electric machines is discussed. However, in any chapter where an individual electric machine is discussed in detail, an equivalent circuit model is used to predict the torque and other performance characteristics. This approach is simple and easily understood.

The dc machine, the three-phase induction machine, and the three-phase synchronous machine are covered extensively in Chapters 4, 5, and 6, respectively. Classical control and also solid-state control of these machines are discussed in detail. Linear induction motors (LIM) and linear synchronous motors (LSM), currently popular for application in transportation systems, are presented. Both voltage source and current source equivalent circuits for the operation of a synchronous machine are used to predict its performance. Operation of self-controlled synchronous motors for use in variable-speed drive systems is discussed. Inverter control of induction machines and the effects of time and space harmonics on induction motor operation are discussed with examples.

Comprehensive coverage of fractional horsepower single-phase motors, widely used in household and office appliances, is presented in Chapter 7. A procedure is outlined for the design of the starting winding of these motors. Special motors such as servomotors, synchro motors, and stepper motors are covered in Chapter 8. These motors play an important role in applications such as position servo systems or computer printers. The transient behavior and the dynamic behavior of the basic machines (dc, induction, and synchronous) are discussed in

Chapter 9. Solid-state converters, needed for solid-state control of various electric machines, are discussed in Chapter 10.

All important aspects of electric machines are covered in this book. In the introduction to each chapter, I indicate the importance of the particular machine covered in that chapter. This is designed to stimulate the reader's interest in that machine and provide motivation to read about it. Following the introduction, I first try to provide a "physical feel" for the behavior of the machine. This is followed by analysis, derivation of the equivalent circuit model, control, application, and so forth.

A large number of worked examples are provided to aid in comprehension of the principles involved.

In present-day industry it is difficult to isolate power electronics technology from electric machines. After graduation, when a student goes into an industry as an engineer, he or she finds that in a motor drive, the motor is just a component of a complex system. Some knowledge of the solid-state control of motors is essential for understanding the functions of the motor drive system. Therefore, in any chapter where an individual motor is discussed, I present controller systems using that particular motor. This is done primarily in a qualitative and schematic manner so that the student can understand the basic operation. In the controller system the solid-state converter, which may be a rectifier, a chopper, or an inverter, is represented as a black box with defined input–output characteristics. The detailed operation of these converters is presented in a separate chapter. It is possible to offer a short course in power electronics based on material covered in Chapter 10 and controller systems discussed in other chapters.

In this book I have attempted to combine traditional areas of electric machinery with more modern areas of control and power electronics. I have presented this in as simple a way as possible, so that the student can grasp the principles without difficulty.

I thank all my undergraduate students who suggested that I write this book and, indeed, all those who have encouraged me in this venture. I acknowledge with gratitude the award of a grant from Queen's University for this purpose. I am thankful to the Dean of the Faculty of Applied Science, Dr. David W. Bacon, and to the Head of the Department of Electrical Engineering, Dr. G. J. M. Aitken, for their support and encouragement. I thank my colleagues in the power area—Drs. Jim A. Bennett, Graham E. Dawson, Tony R. Eastham, and Vilayil I. John—with whom I discussed electric machines while teaching courses on this subject. I thank Mr. Rabin Chatterjee, with whom I discussed certain sections of the manuscript. I am grateful to my graduate students, Chandra Namuduri, Eddy Ho, and Pradeep Nandam, for their assistance. Pradeep did the painful job of proofreading the final manuscript. I thank our administrative assistant, Mr. Perry Conrad, who supervised the typing of the manuscript. I thank the departmental secretaries, Sheila George, Marlene Hawkey, Marian Rose, Kendra Pople-Easton, and Jessie Griffin, for typing the manuscript at various stages. I express my profound gratitude to Chuck (Prof. C. H. R. Campling), who spent many hours reading and correcting the text. His valuable counseling and continued encouragement throughout have made it possible for me to complete this book. Finally, I appreciate the patience and solid support of my family—my wife, Maya, and my enthusiastic children, Sujit, Priya, and Debashis, who could hardly wait to have a copy of the book presented to them so that they could show it to their friends.

Queen's University P. C. SEN
Kingston, Ontario, Canada
April 1987

CONTENTS

chapter one

MAGNETIC CIRCUITS

This book is concerned primarily with the study of devices that convert electrical energy into mechanical energy or the reverse. Rotating electrical machines, such as dc machines, induction machines, and synchronous machines, are the most important ones used to perform this energy conversion. The transformer, although not an electromechanical converter, plays an important role in the conversion process. Other devices, such as actuators, solenoids, and relays, are concerned with linear motion. In all these devices, magnetic materials are used to shape and direct the magnetic fields that act as a medium in the energy conversion process. A major advantage of using magnetic material in electrical machines is the fact that high flux density can be obtained in the machine, which results in large torque or large machine output per unit machine volume. In other words, the size of the machine is greatly reduced by the use of magnetic materials.

In view of the fact that magnetic materials form a major part in the construction of electric machines, in this chapter properties of magnetic materials are discussed and some methods for analyzing the magnetic circuits are outlined.

1.1 MAGNETIC CIRCUITS

In electrical machines, the magnetic circuits may be formed by ferromagnetic materials only (as in transformers) or by ferromagnetic materials in conjunction with an air medium (as in rotating machines). In most electrical machines, except permanent magnet machines, the magnetic field (or flux) is produced by passing an electrical current through coils wound on ferromagnetic materials.

1.1.1 i–H RELATION

We shall first study how the current in a coil is related to the magnetic field intensity (or flux) it produces. When a conductor carries current, a magnetic field is produced around it, as shown in Fig. 1.1. The direction of flux lines or magnetic field intensity H can be determined by what is known as the *thumb rule*, which states that if the conductor is held with the right

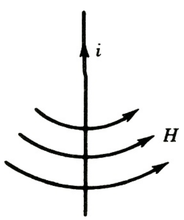

FIGURE 1.1 Magnetic field around a current-carrying conductor.

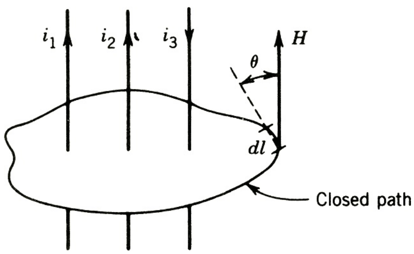

FIGURE 1.2 Illustration of Ampère's circuit law.

hand with the thumb indicating the direction of current in the conductor, then the fingertips will indicate the direction of magnetic field intensity. The relationship between current and field intensity can be obtained by using *Ampère's circuit law*, which states that the line integral of the magnetic field intensity H around a closed path is equal to the total current linked by the contour.

Referring to Fig. 1.2,

$$\oint H \cdot dl = \sum i = i_1 + i_2 - i_3 \tag{1.1}$$

where H is the magnetic field intensity at a point on the contour and dl is the incremental length at that point. If θ is the angle between vectors H and dl, then

$$\oint H \cdot dl \cos\theta = \sum i \tag{1.2}$$

Now, consider a conductor carrying current i as shown in Fig. 1.3. To obtain an expression for the magnetic field intensity H at a distance r from the conductor, draw a circle of radius r. At each point on this circular contour, H and dl are in the same direction, that is, $\theta = 0$. Because of symmetry, H will be the same at all points on this contour. Therefore, from Eq. 1.2,

$$\oint H \cdot dl = i$$

$$H\,2\pi r = i$$

$$H = \frac{i}{2\pi r} \tag{1.2a}$$

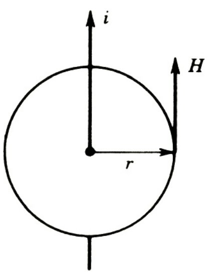

FIGURE 1.3 Determination of magnetic field intensity H due to a current-carrying conductor.

1.1.2 *B–H* RELATION

The magnetic field intensity H produces a magnetic flux density B everywhere it exists. These quantities are functionally related by

$$B = \mu H \text{ weber/m}^2 \quad \text{or} \quad \text{tesla} \tag{1.3}$$

$$B = \mu_r \mu_0 H \text{ Wb/m}^2 \quad \text{or} \quad \text{T} \tag{1.4}$$

where μ is a characteristic of the medium and is called the *permeability* of the medium
μ_0 is the permeability of free space and is $4\pi 10^{-7}$ henry/meter
μ_r is the *relative permeability* of the medium

For free space or electrical conductors (such as aluminum or copper) or insulators, the value of μ_r is unity. However, for ferromagnetic materials such as iron, cobalt, and nickel, the value of μ_r varies from several hundred to several thousand. For materials used in electrical machines, μ_r varies in the range of 2000 to 6000. A large value of μ_r implies that a small current can produce a large flux density in the machine.

1.1.3 MAGNETIC EQUIVALENT CIRCUIT

Figure 1.4 shows a simple magnetic circuit having a ring-shaped magnetic core, called a *toroid*, and a coil that extends around the entire circumference. When current i flows through the coil of N turns, magnetic flux is mostly confined in the core material. The flux outside the toroid, called *leakage flux*, is so small that for all practical purposes it can be neglected.

Consider a path at a radius r. The magnetic intensity on this path is H and, from Ampère's circuit law,

$$\oint H \cdot dl = Ni \tag{1.5}$$

$$Hl = Ni \tag{1.5a}$$

$$H\, 2\pi r = Ni \tag{1.6}$$

The quantity Ni is called the *magnetomotive force* (*mmf*) F, and its unit is ampere-turn.

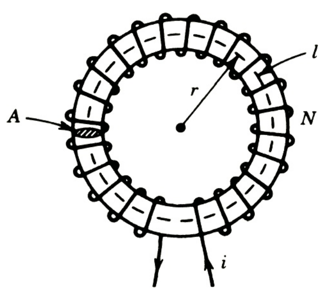

FIGURE 1.4 Toroid magnetic circuit.

$$Hl = Ni = F \tag{1.7}$$

$$H = \frac{N}{l} i \text{ At/m} \tag{1.8}$$

From Eqs. 1.3 and 1.8

$$B = \frac{\mu Ni}{l} \text{ T} \tag{1.9}$$

If we assume that all the fluxes are confined in the toroid—that is, there is no magnetic leakage—the flux crossing the cross section of the toroid is

$$\Phi = \int B\, dA \tag{1.10}$$

$$\Phi = BA \text{ Wb} \tag{1.11}$$

where B is the average flux density in the core and A is the area of cross section of the toroid. The average flux density may correspond to the path at the mean radius of the toroid. If H is the magnetic intensity for this path, then from Eqs. 1.9 and 1.11,

$$\begin{aligned} \Phi &= \frac{\mu Ni}{l} A = \frac{Ni}{l/\mu A} \\ &= \frac{Ni}{\mathscr{R}} \end{aligned} \tag{1.12}$$

$$= \frac{F}{\mathscr{R}} \tag{1.13}$$

where

$$\mathscr{R} = \frac{l}{\mu A} = \frac{1}{P} \tag{1.14}$$

is called the *reluctance* of the magnetic path, and P is called the *permeance* of the magnetic path. Equations 1.12 and 1.13 suggest that the driving force in the magnetic circuit of Fig. 1.4 is the magnetomotive force F $(=Ni)$, which produces a flux Φ against a magnetic reluctance $\mathscr{R}$. The magnetic circuit of the toroid can therefore be represented by a magnetic equivalent circuit as shown in Fig. 1.5*a*. Also note that Eq. 1.13 has the form of Ohm's law for an electric

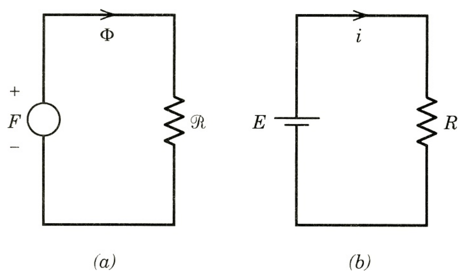

FIGURE 1.5 Analogy between (a) magnetic circuit and (b) electric circuit.

TABLE 1.1 Electrical versus Magnetic Circuits

	Electric Circuit	Magnetic Circuit
Driving force	Emf (E)	Mmf (F)
Produces	Current ($i = E/R$)	Flux ($\Phi = F/\mathscr{R}$)
Limited by	Resistance ($R = l/\sigma A$)[a]	Reluctance ($\mathscr{R} = l/\mu A$)[a]

[a] σ, Conductivity; μ, permeability.

circuit ($i = E/R$). The analogous electrical circuit is shown in Fig. 1.5*b*. A magnetic circuit is often looked upon as analogous to an electric circuit. The analogy is illustrated in Table 1.1.

1.1.4 MAGNETIZATION CURVE

If the magnetic intensity in the core of Fig. 1.4 is increased by increasing current, the flux density in the core changes in the way shown in Fig. 1.6. The flux density B increases almost linearly in the region of low values of the magnetic intensity H. However, at higher values of H, the change of B is nonlinear. The magnetic material shows the effect of saturation. The $B-H$ curve, shown in Fig. 1.6, is called the *magnetization curve*. The reluctance of the magnetic path is dependent on the flux density. It is low when B is low, and high when B is high. The magnetic circuit differs from the electric circuit in this respect; resistance is normally independent of current in an electric circuit, whereas reluctance depends on the flux density in the magnetic circuit.

The $B-H$ characteristics of three different types of magnetic cores—cast iron, cast steel, and silicon sheet steel—are shown in Fig. 1.7. Note that to establish a certain level of flux density B^* in the various magnetic materials, the values of current required are different.

1.1.5 MAGNETIC CIRCUIT WITH AIR GAP

In electric machines, the rotor is physically isolated from the stator by the air gap. A cross-sectional view of a dc machine is shown in Fig. 1.8. Practically the same flux is present in the poles (made of magnetic core) and the air gap. To maintain the same flux density, the air gap will require much more mmf than the core. If the flux density is high, the core portion of the magnetic circuit may exhibit a saturation effect. However, the air gap remains unsaturated, since the $B-H$ curve for the air medium is linear (μ is constant).

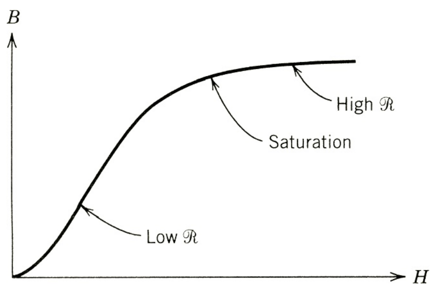

FIGURE 1.6 $B-H$ characteristic (magnetization curve).

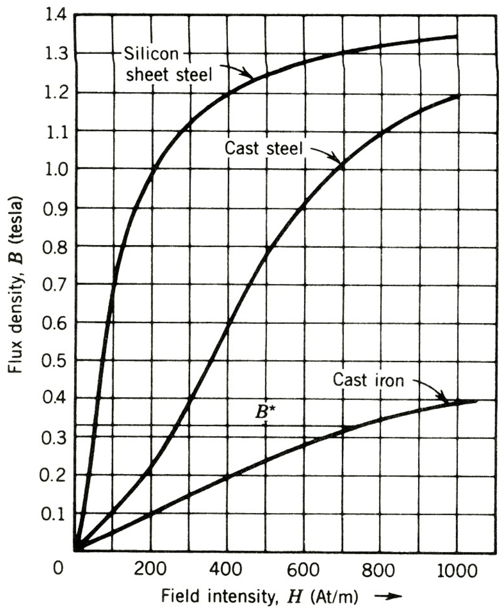

FIGURE 1.7 Magnetization curves.

A magnetic circuit having two or more media—such as the magnetic core and air gap in Fig. 1.8—is known as a *composite structure*. For the purpose of analysis, a magnetic equivalent circuit can be derived for the composite structure.

Let us consider the simple composite structure of Fig. 1.9*a*. The driving force in this magnetic circuit is the mmf, $F = Ni$, and the core medium and the air gap medium can be represented by their corresponding reluctances. The equivalent magnetic circuit is shown in Fig. 1.9*b*.

$$\mathcal{R}_c = \frac{l_c}{\mu_c A_c} \tag{1.15}$$

$$\mathcal{R}_g = \frac{l_g}{\mu_0 A_g} \tag{1.16}$$

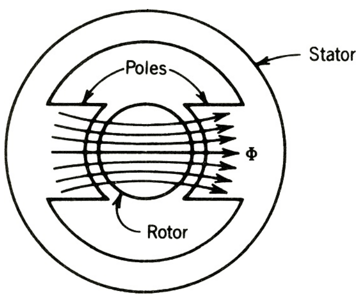

FIGURE 1.8 Cross section of a rotating machine.

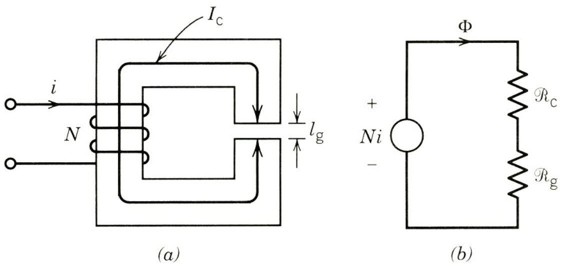

FIGURE 1.9 Composite structure. (*a*) Magnetic core with air gap. (*b*) Magnetic equivalent circuit.

$$\Phi = \frac{Ni}{\mathscr{R}_c + \mathscr{R}_g} \tag{1.17}$$

$$Ni = H_c l_c + H_g l_g \tag{1.18}$$

where l_c is the mean length of the core
l_g is the length of the air gap

The flux densities are

$$B_c = \frac{\Phi_c}{A_c} \tag{1.19}$$

$$B_g = \frac{\Phi_g}{A_g} \tag{1.20}$$

In the air gap the magnetic flux lines bulge outward somewhat, as shown in Fig. 1.10; this is known as *fringing* of the flux. The effect of the fringing is to increase the cross-sectional area of the air gap. For small air gaps the fringing effect can be neglected. If the fringing effect is neglected, the cross-sectional areas of the core and the air gap are the same and therefore

$$A_g = A_c$$

$$B_g = B_c = \frac{\Phi}{A_c}$$

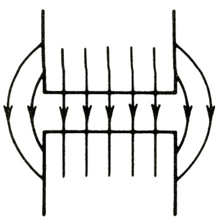

FIGURE 1.10 Fringing flux.

EXAMPLE 1.1

Figure E1.1 represents the magnetic circuit of a primitive relay. The coil has 500 turns and the mean core path is $l_c = 360$ mm. When the air gap lengths are 1.5 mm each, a flux density of 0.8 tesla is required to actuate the relay. The core is cast steel.

(a) Find the current in the coil.

(b) Compute the values of permeability and relative permeability of the core.

(c) If the air gap is zero, find the current in the coil for the same flux density (0.8 T) in the core.

Solution

(a) The air gap is small, so fringing can be neglected. Hence the flux density is the same in both air gap and core. From the $B-H$ curve of the cast steel core (Fig. 1.7).

For

$$B_c = 0.8 \text{ T}, \quad H_c = 510 \text{ At/m}$$

$$\text{mmf } F_c = H_c l_c = 510 \times 0.36 = 184 \text{ At}$$

For the air gap,

$$\text{mmf } F_g = H_g 2l_g = \frac{B_g}{\mu_0} 2l_g = \frac{0.8}{4\pi 10^{-7}} \times 2 \times 1.5 \times 10^{-3}$$

$$= 1910 \text{ At}$$

Total mmf required:

$$F = F_c + F_g = 184 + 1910 = 2094 \text{ At}$$

Current required:

$$i = \frac{F}{N} = \frac{2094}{500} = 4.19 \text{ amps}$$

Note that although the air gap is very small compared to the length of the core ($l_g = 1.5$ mm, $l_c = 360$ mm), most of the mmf is used at the air gap.

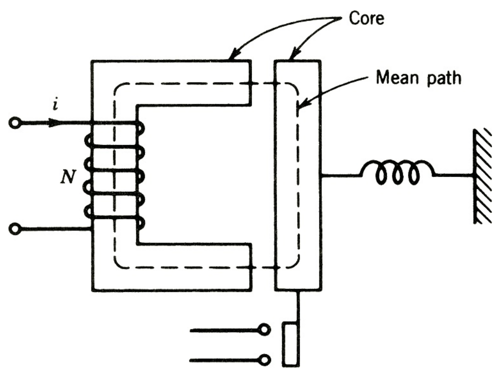

FIGURE E1.1 $N = 500$ turns, $l_c = 36$ cm.

(b) Permeability of core:

$$\mu_c = \frac{B_c}{H_c} = \frac{0.8}{510} = 1.57 \times 10^{-3}$$

(c) Relative permeability of core:

$$\mu_r = \frac{\mu_c}{\mu_0} = \frac{1.57 \times 10^{-3}}{4\pi 10^{-7}} = 1250$$

$$F = H_c l_c = 510 \times 0.36 = 184 \text{ At}$$

$$i = \frac{184}{500} = 0.368 \text{ A}$$

Note that if the air gap is not present, a much smaller current is required to establish the same flux density in the magnetic circuit. ■

EXAMPLE 1.2

Consider the magnetic system of Example 1.1. If the coil current is 4 amps when each air gap length is 1 mm, find the flux density in the air gap.

Solution

In Example 1.1, the flux density was given and so it was easy to find the magnetic intensity and finally the mmf. In this example, current (or mmf) is given and we have to find the flux density. The $B-H$ characteristic for the air gap is linear, whereas that of the core is nonlinear. We need nonlinear magnetic circuit analysis to find out the flux density. Two methods will be discussed.

1. *Load line method*. For a magnetic circuit with core length l_c and air gap length l_g,

$$Ni = H_g l_g + H_c l_c = \frac{B_g}{\mu_0} l_g + H_c l_c$$

Rearranging,

$$B_g = B_c = -\mu_0 \frac{l_c}{l_g} H_c + \frac{Ni\mu_0}{l_g} \tag{1.21}$$

This is in the form $y = mx + c$, which represents a straight line. This straight line (also called the *load line*) can be plotted on the $B-H$ curve of the core. The slope is

$$m = -\mu_0 \frac{l_c}{l_g} = -4\pi 10^{-7} \frac{360}{2} = -2.26 \times 10^{-4}$$

The intersection on the B axis is

$$c = \frac{Ni\mu_0}{l_g} = \frac{500 \times 4 \times 4\pi 10^{-7}}{2 \times 10^{-3}} = 1.256 \text{ tesla}$$